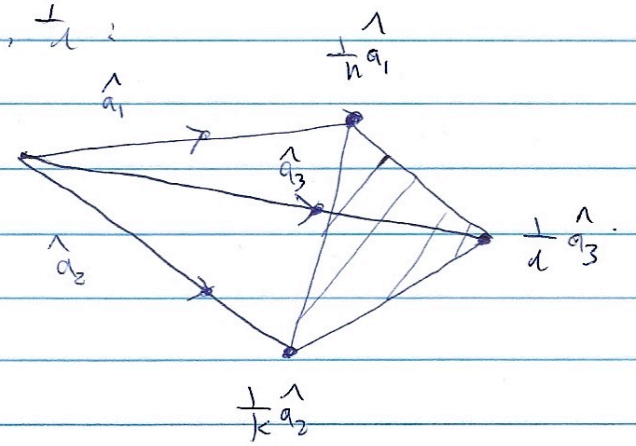
Kittel sstate

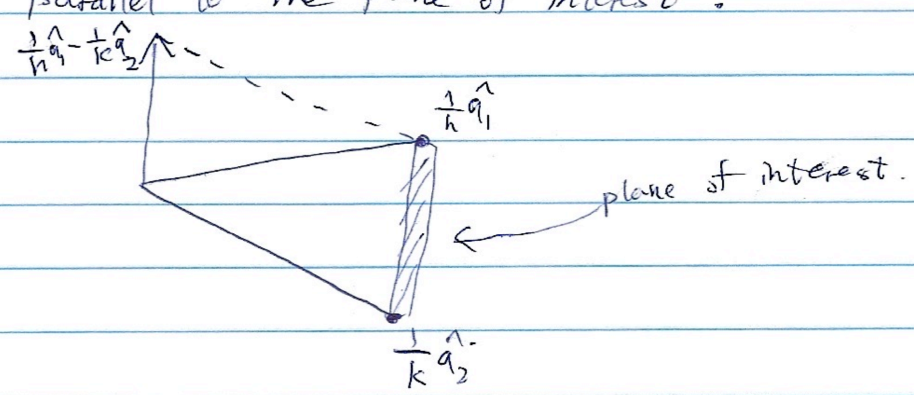
2.1 (a) (correction).

We first find parameterization of the plane (hkl).

Consider a plane that intersects the primitive vectors $\hat{a}_1, \hat{a}_2, \hat{a}_3$ at $\frac{1}{h}, \frac{1}{k}, \frac{1}{l}$:



This plane is spanned by 2 linearly-independent vectors. To find those vectors consider the vector $\frac{1}{h}\hat{a}_1 - \frac{1}{k}\hat{a}_2$, it's simple to view in the plane of $\hat{a}_1, \hat{a}_2$ that this ~~plane~~ vector is parallel to the plane of interest:



Similarly, $\frac{1}{k}\hat{a}_2 - \frac{1}{l}\hat{a}_3$ will be parallel to the plane of interest as well. Since these two vectors are linearly independent, they span the plane of interest and we can write for any vector on the plane

$$x\left(\frac{1}{h}\hat{a}_1 - \frac{1}{k}\hat{a}_2\right) + y\left(\frac{1}{k}\hat{a}_2 - \frac{1}{l}\hat{a}_3\right)$$

$$= \boxed{\frac{x}{h}\hat{a}_1 + \frac{y-x}{k}\hat{a}_2 - \frac{y}{l}\hat{a}_3}$$

Now consider the product of the preceeding set of vectors parameterized by x, y with the vector $\vec{G} = h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3$:

$$\left(\frac{x}{h}\hat{a}_1 + \frac{y-x}{k}\hat{a}_2 - \frac{y}{l}\hat{a}_3\right)\cdot\left(h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3\right)$$

by definition, $\vec{b}_i = 2\pi \dfrac{\hat{a}_j \times \hat{a}_k}{\hat{a}_1 \cdot \hat{a}_2 \times \hat{a}_3}$ following cyclic perm.

~~Since~~ Thus $\hat{a}_i \cdot \vec{b}_j = \dfrac{\hat{a}_i \cdot \hat{a}_j \times \hat{a}_k}{\hat{a}_1 \cdot \hat{a}_2 \times \hat{a}_3}(2\pi) = (2\pi)\,\delta_{ij}$

$\Rightarrow$ The above dot product can be evaluated as.

$$(2\pi)\left[\frac{x}{h}h + \frac{y-x}{k}k + \left(-\frac{y}{l}\right)l\right]$$

$$= \boxed{0}.$$

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